9PR2.7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.

Multiplying and Dividing Polynomials Pictorially and Algebraically

Polynomials can be multiplied and divided using algebra tiles or pencil and paper.

Multiplying and Dividing Polynomials with Algebra Tiles

Multiplying polynomials using algebra tiles is similar to finding the area of a rectangle.

To multiply polynomials with algebra tiles, follow these steps:

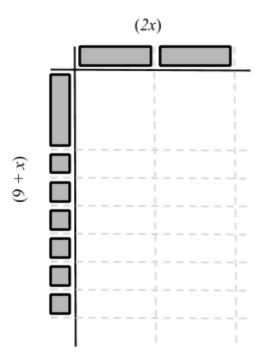
- 1. Set up the grid.
- 2. Multiply the vertical tiles outside the grid by the horizontal tiles outside the grid.
- 3. Add the like terms inside the grid to identify the solution.

Example

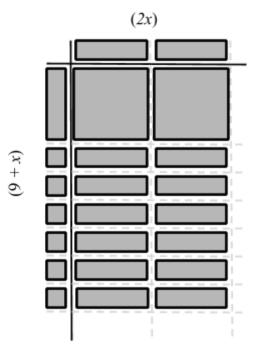
Expand and simplify 2x(x+6). Write the polynomial that represents the area.

Solution

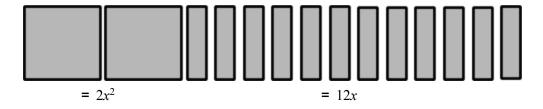
Step 1 Set up the grid.



Step 2 Multiply the edge of the vertical tile by the edge of the horizontal tile to get the area.



Step 3 Add the like terms inside the grid to identify the solution.



The answer is $2x^2 + 12x$.

Remember these rules for performing multiplication or division with negatives:

- Two positive or two negatives equal a positive or shaded value.
- A positive and a negative equal a negative or unshaded value.

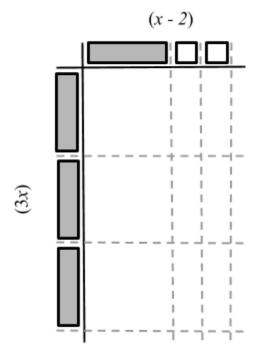
Example

Expand and simplify 3x(x-2) using algebra tiles.

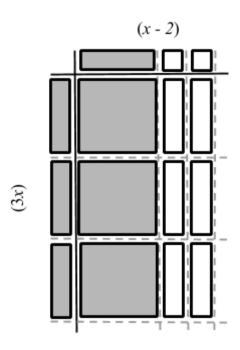
Solution

Step 1

Set up the grid. Adding the dotted lines shows the shape of the tile that is the product of the two outside tiles.



Step 2Multiply the edge of the vertical tile by the edge of the horizontal tile to get the area.



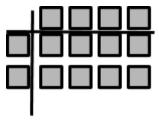
Step 3

Add the like terms inside the grid to identify the solution

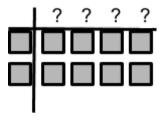


$$3x(x-2) = 3x^2 - 6x$$

When multiplying polynomials using algebra tiles, the solution is found by determining the area of a rectangle. For example, 4×2 .



When dividing polynomials using algebra tiles, the area of a rectangle and length of one side is given. The length of the second side needs to be determined. For example, $8 \div 2$.



To divide polynomials with algebra tiles, follow these steps:

- 1) Set up the multiplication grid with the divisor on the side.
- 2) Arrange the dividend into a rectangle.
- 3) Determine the quotient.

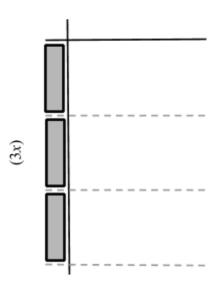
Example

Use algebra tiles to find the quotient of the expression $\frac{3x^2+9x}{3x}$.

Solution

Step 1

Set up the grid with the divisor on the side.

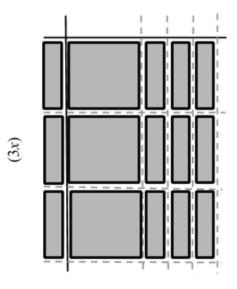


Step 2

Arrange the dividend into the rectangle. There are a few rules for how the tiles are displayed in the rectangle.

• Only equal length sides may touch.

- Big squares cannot touch little squares.
- Little Squares must all be together.

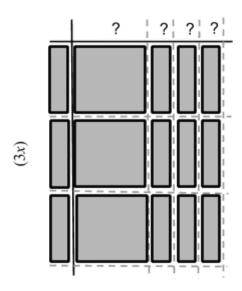


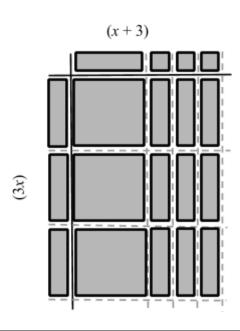
Step 3

Determine the quotient.

The quotient is the side length of each of the tiles

$$(3x^2 + 9x) \div 3x = x + 3$$





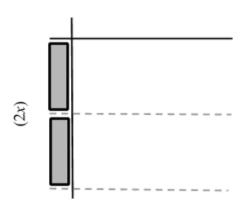
Example

Use algebra tiles to find the quotient of the expression $(2x^2 - 4x) \div (2x)$.

Solution

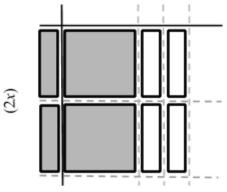
Step 1

Set up the grid with the divisor on the side.

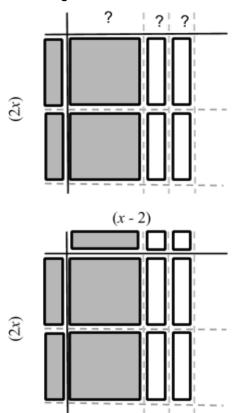


Arrange the dividend into the rectangle. There are a few rules for how the tiles are displayed in the rectangle.

- Only equal length sides may touch.
- Big squares cannot touch little squares.
- Little Squares must all be together.



Step 3Determine the quotient. The quotient is the side length of each of the tiles.



$$(2x^2 - 4x) \div (2x) = (x - 2)$$

Multiplying and Dividing Polynomials Algebraically

To multiply polynomials, apply the distributive property. Multiply the numerical coefficients together. Then, multiply the variables together by adding the exponents of the variables with the same base.

Example

Expand and simplify $3(2x^2-6x+4)$.

Solution

To expand an expression means to multiply each term inside the brackets by the term outside the brackets.

Step 1

Distribute the term outside the brackets to each term inside the brackets.

$$(3)(2x^2) + (3)(-6x) + (3)(4)$$

Notice the addition signs are written between each group being multiplied. Minus six (-6) is written negative six (-6) in the expansion when applying the distributive property.

Step 2

Multiply each pair of terms. Multiply the coefficients. Add the exponents of like bases.

$$= (3 \times 2)(x^2) + (3 \times -6)(x) + (3)(4)$$

$$=6x^2-18x+12$$

When dividing a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example

Find the quotient of $\frac{4x^3+8x^2-16x}{2x}$

Solution

Step 1

Break the polynomial into its terms. The polynomial is the expression with one or more terms.

$$4x^3$$
, $8x^2$, $-16x$

Step 2

Divide each term by the divisor. The divisor is 2x.

Divide the coefficients. Subtract the exponents of the powers with the same base.

$$= \frac{4x^3}{2x} + \frac{8x^2}{2x} + \frac{-16x}{2x}$$
$$= (4 \div 2)(x^{3-1}) + (8 \div 2)(x^{2-1}) - (16 \div 2)(x^{1-1})$$

Step 3

Write the quotient, or answer.

$$=2x^2+4x-8$$