9SS3.5 Demonstrate an understanding of line and rotation symmetry.

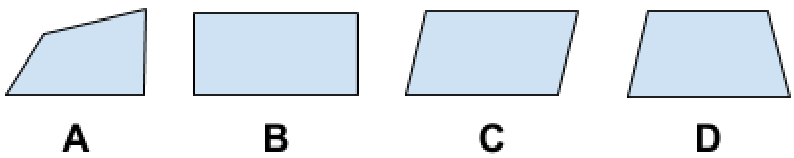
Line Symmetry and Rotational Symmetry

A transformation that results in the same image as the original shape is called a symmetry operation. There are two kinds of symmetry: line symmetry and rotational symmetry.

Line of Symmetry

A **line of symmetry** is a line that divides a figure into two identical parts. A shape of an object is symmetrical if both sides are the same when a line is drawn through the middle of it.

Example



Sort the given quadrilaterals by their number of lines of symmetry, and explain the groups used.

Solution

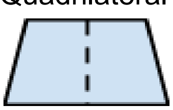
The quadrilaterals can be sorted into three groups.

Group 1: No lines of symmetry

Quadrilaterals A and C have no lines of symmetry.

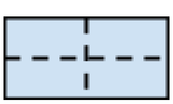
Group 2: One line of symmetry

Quadrilateral D has one line of symmetry.



Group 3: More than one line of symmetry

Quadrilateral B has two lines of symmetry



When given one-half of a 2-D shape, it is possible to complete the entire shape by drawing the identical image on the other side of the line of symmetry.

A translation is a transformation that slides a shape in a certain direction across a fixed distance. The original shape and its image are the same shape and size. When a 2-D shape undergoes a translation, there is no symetry between the original shape and its image.

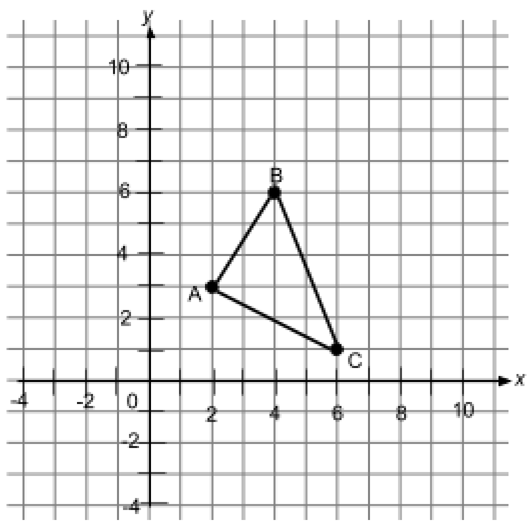
Example

Draw ΔABC with vertices *A*(2, 3), B(4, 6), and C(6, 1). Translate the triangle 4 units left and 3 units down. Label the translated image A', B', C'.

Solution

Step 1

Draw the original shape on the Cartesian plane. Plot and label each point as given in the question, then connect the points with line segments.



Step 2

Add or subtract, from the original coordinates of the vertices, the number of units the image will move horizontally or vertically.

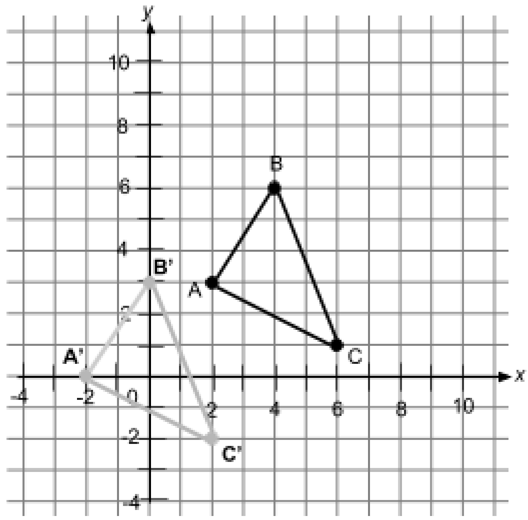
For each point, subtract 4 from the x-coordinate (4 to the left) and 3 from the y-coordinate (3 down)

$$A'(2-4,\ 3-3) = (-2,\ 0)$$
$$B'(4-4,\ 6-3) = (0,\ 3)$$
$$C'(6-4,\ 1-3) = (2,\ -2)$$

Step 3

Draw the translated image on the cartesian plane.

Plot and label each new point, then connect the new points with line segments to form the translated triangle, $\Delta A'B'C'$



In a reflection, often referred to as a flip, the shape is mirrored across a reflection line. Think of the reflection line as the line that is folded to create a mirror image on the other side of the line. The coordinates of a reflected image are the same distance away from the mirror, or reflecting line, as in the original shape. The reflection image is on the opposite side of the reflection line from the original shape. There is a line of symmetry between the original shape and the image.

Example

Draw ΔABC with vertices $A(-5,\ -2)$, $B(-3,\ -1)$, and $C(-3,\ -3)$. Then, draw the reflection image using a line of symmetry (reflection line) one unit above the x-axis.

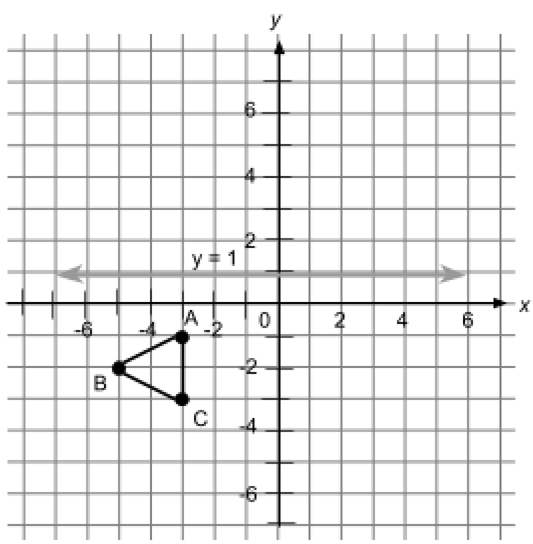
The equation of the reflection line is $y = 1$ **(This will be important in Unit 4)**

Label the image with single primes on the letters.

Solution

Step 1

Draw the original shape on the Cartesian plane. Plot and label each point as given in the question. Then, connect the points with line segments. Add the reflection line.



Step 2

Determine the coordinates of the reflected image. The image is reflected about the x-axis. This means the x-coordinates will remain the same while the y-coordinates change.

Calculate the distance each vertex is from the reflection line.

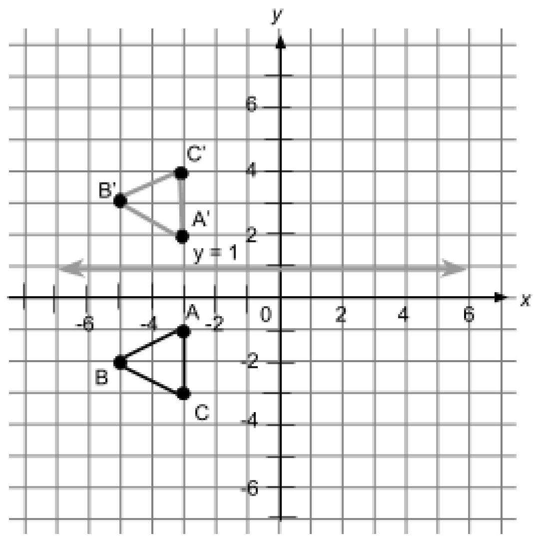
Since A is 3 units below the reflection line, A' will be 3 unit above the reflection line. $A'(-5,\ 4)$

Since B is 2 units below the reflection line, B' will be 2 units above the reflection line. $B'(-3,\ 3)$

Since C is 4 units below the reflection line, C' will be 4 units above the reflection line. $C'(-3,\ 5)$

Step 3

Draw the reflected image on the Cartesian plane. Plot and label each new point with a prime on the letter to indicate that it is the image of the original shape. Then, connect the points with line segments.



Rotational Symmetry

When a shape coincides with itself after a rotation of less than 360° about its center, it is said to have **rotational symmetry**.

The number of times a shape returns to its original position during a rotation of 360° is called the **order of rotation**.

The relationship between the **angle of rotation** and the order of rotation is illustrated in the following formula:

$$\textit{angle of rotation} = \frac{360^o}{\textit{order of rotation}}$$

Example

Determine the order of rotational symmetry and the angle of rotation of a regular pentagon.

Solution

Step 1

Determine the order of rotational symmetry. Since a regular pentagon has 5 equal sides and 5 lines of symmetry, its order of rotational symmetry will also be 5.

The rotational symmetry means that the pentagon can be rotated 5 times before it reaches its original position. It will look exactly like the first position each time it is rotated.

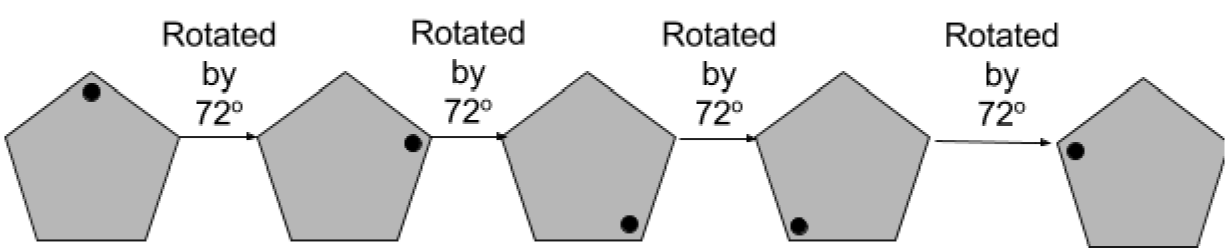
Step 2

Calculate the angle of rotation.

Apply the following formula:

$$\begin{aligned}\textit{angle of rotation} &= \frac{360^o}{\textit{order of rotation}}\\ &= \frac{360^o}{5}\\ &= 72^o\end{aligned}$$

This set of diagrams shows how a pentagon can be rotated until it reaches its first position. The dot on the pentagon shows how the pentagon rotates 72° each time.



A rotation, sometimes referred to as a turn, is a transformation that rotates a shape about a point. The original shape and its image are the same shape and size. When a 2-D shape undergoes a rotation, there is rotational symmetry between the original shape and its image.

Example

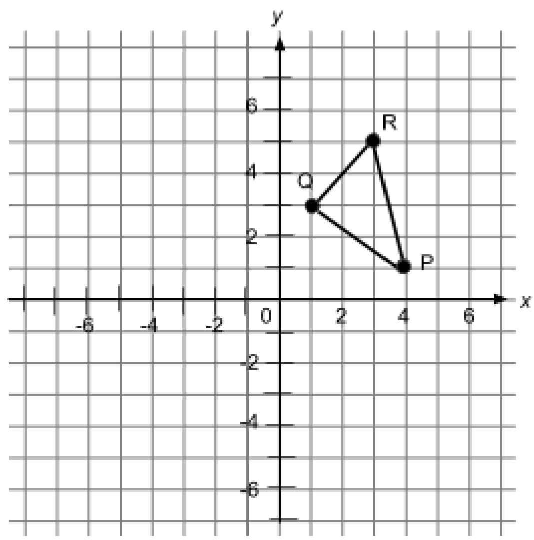
On Cartesian plane, triangle PQR has vertices of $P(4,\ 1)$, $Q(1,\ 3)$, and $R(3,\ 5)$.

Draw triangles PQR and $P'Q'R'$ after a rotation of 180° counterclockwise about point P.

Solution

Step 1

Draw the original shape on a Cartesian plane. Plot and label each point as given. Then, connect the points with line segments.



Step 2

Trace the original shape, and rotate it as directed.

When rotating an image 180°, direction does not matter because clockwise and counterclockwise rotations have the same result.

Make sure the traced shape is directly on top of the original shape. Place the pencil on P, and turn the tracing paper 180°, or a ½ turn to the right or left.

Step 3

Draw the rotated image on the Cartesian plane. Plot and label each new point with a prime on the letter to indicate that it is the image of the original shape. Then, connect the points with line segments.