9SS1.1 Solve problems and justify the solution strategy, using circle properties.

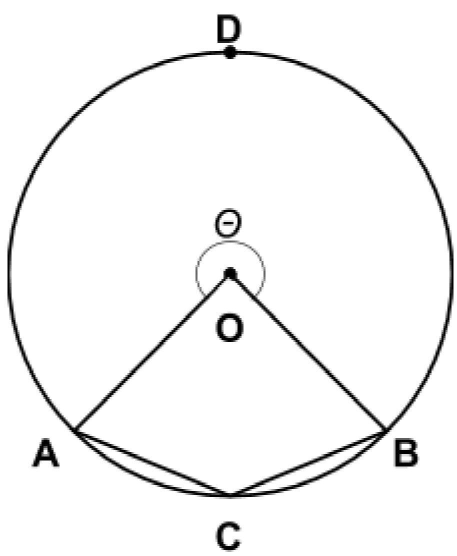
Problem Solving Using Circle Properties

There are four properties of circles:

- A perpendicular from the center of a circle to a chord bisects the chord, The perpendicular divides the chord into two equal parts.
- The measure of the central angle is twice the measure of the inscribed angle subtended by the same arc.
- Inscribed angles subtended by the same arc (or chord) are congruent (equal).
- A tangent to a circle is perpendicular to the radius at the point of tangency.

These circle properties can be applied to solve problems involving circles.

Example 1



In the given diagram, ∠ACD = 165º

What is the measure of angle Θ? ______________º

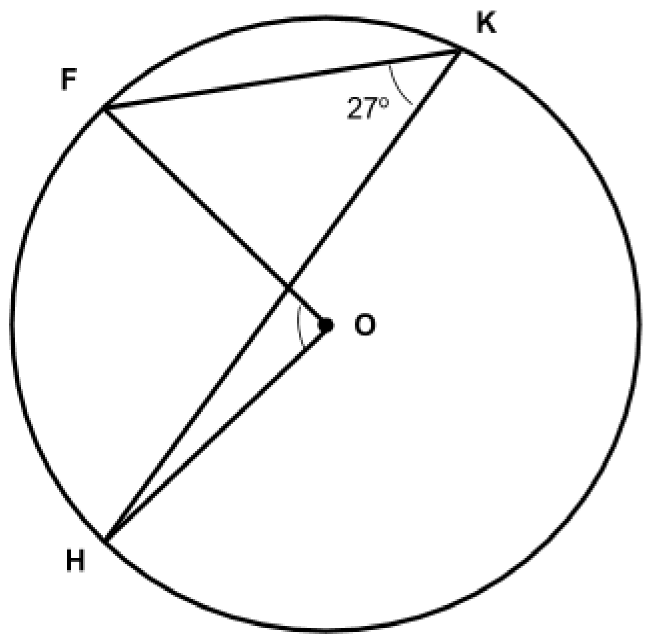
Solution

Step 1
Interpret the diagram. Apply properties of inscribed and central angles.

Angle Θ (reflex angle AOB) is the central angle subtended by major arc ADB. Angle ACB is an inscribed angle also subtended by major arc ADB. The measure of angle Θ is twice the measure of inscribed angle ACB.

Step 2
Calculate the measure of Θ.
Θ = 2∠ACD
Θ = 2(165º)
Θ = 330º

Example 2



Determine the measure of central angle FOH.

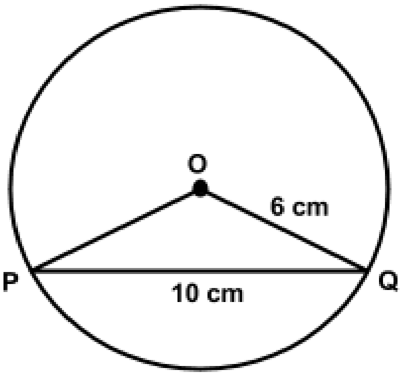
Solution

Step 1
Central angle FOH and inscribed angle FKH are subtended by the same arc.

Step 2
Apply properties of central and inscribed angles. The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.

$\angle FOH = 2\angle FKH$
$\angle FOH = 2(27^\circ)$
$\angle FOH = 54^\circ$

Example 3



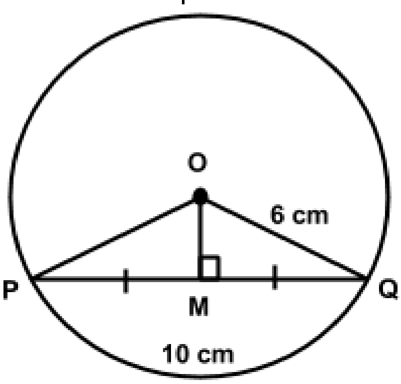
A circle with center O has a chord PQ that is 10 cm in length.

If a radius of the circle is 6 cm, then what is the area of triangle POQ correct to the nearest tenth of a centimeter?

Solution

Step 1

Draw segment OM from the center of the circle to the midpoint of chord PW, and determine the length of MW.



According to the perpendicular bisector theorem, a segment from the center of the circle to the midpoint of a chord is perpendicular to that chord. Since the length of segment MQ is equal to the length of segment MP, the length of MQ is equal to half the length of PQ.

$$MQ = \frac{PQ}{2}$$
$$= \frac{10}{2}$$
$$= 5 \text{ cm}$$

Step 2

Calculate the height of the triangle. Use the right triangle OMQ and the Pythagorean theorem to determine the length of OM.

$$c^2 = a^2 + b^2$$
$$(OQ)^2 = (OM)^2 + (MQ)^2$$
$$(OM)^2 = (OQ)^2 - (MQ)^2$$
$$(OM)^2 = 6^2 - 5^2$$
$$(OM)^2 = 36 - 25$$
$$\sqrt{(OM)^2} = \sqrt{36 - 25}$$
$$OM = \sqrt{11cm}$$

Step 3

Determine the area of triangle OPQ. Apply the area formula for the triangle, substitute in the known values, and evaluate.

$$A = \frac{1}{2}\,bh$$
$$A = \frac{1}{2}\,(PQ)(OM)$$
$$A = \frac{1}{2}\,(10)(\sqrt{11})$$
$$A = 5\sqrt{11}$$
$$A = 16.6\ \text{cm}^2$$