9N1.5 Determine the square root of positive rational numbers that are perfect squares.

Square Roots and Perfect Squares

The **square root** of a given number is a value that when multiplied by itself results in the given number. For example, the square root of 9 is 3 because $3 \times 3 = 9$.
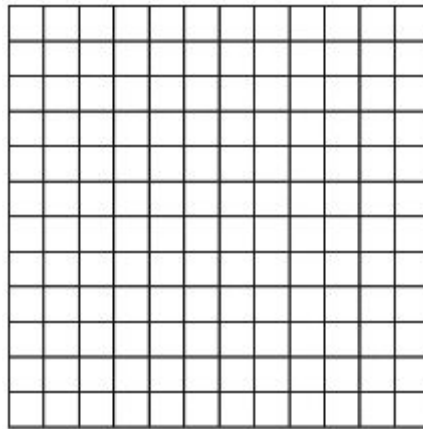
Example
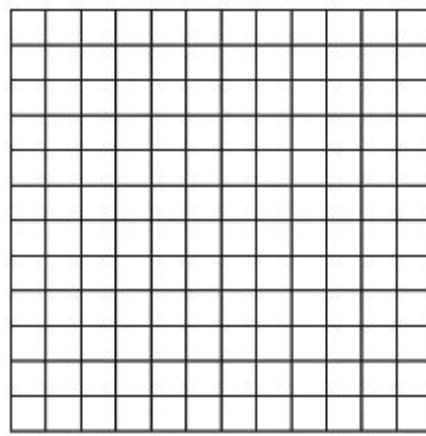Use a diagram to determine the square root of 121.

Solution
Step 1
Draw a square with an area of 121 units.

Step 2
Count the number of units along one side of the square.

$\sqrt{121} = 11$ units

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The square root of 121 is 11. This can also be written as $\sqrt{121} = 11$.

A number that has a whole number as its square root is called a **perfect square**. Perfect squares have the unique characteristic of having an odd number of factors.

Example
Given the numbers 81, 24, 102, 144, identify the perfect squares by ordering their factors from smallest to largest.

Solution
The square root of each perfect square is bolded.

Factors of 81: 1, 3, **9,** 27, 81

Since there are an odd number of factors, 81 is a perfect square.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Since there are an even number of factors, 24 is not a perfect square.

Factors of 102: 1, 2, 3, 6, 17, 34, 51, 102

Since there are an even number of factors, 102 is not a perfect square.

Factors of 144: 1, 2, 3, 4, 6, 8, 9, **12,** 16, 18, 24, 36, 48, 72, 144

Since there are an odd number of factors, 144 is a perfect square.

Rational numbers include square numbers that are decimal numbers. The rational number must have an even number of digits to the right of the decimal point for it to be a square number.

Example
Given the square root 1.3, what is the square number?

Solution
Step 1

Determine the square number.

A square number is the product of a given number multiplied by itself. In other words, it is the given number squared.

Using a calculator, square 1.3.

| 1 | . | 3 | X | 1 | . | 3 | = |

The product of 1.3^2 is 1.69

Step 2

Verify by calculating the square root of the answer.

Use a calculator to find the square root of 1.69.

$$\boxed{1}\ \boxed{.}\ \boxed{6}\ \boxed{9}\ \boxed{\sqrt{}}\ \boxed{=}$$

The square root of 1.69 is 1.3.

The positive rational number is 1.69.

Use the following relationships to check for errors involving roots:

$\sqrt{a} = b$ and $b^2 = a$

Example

Is the calculation $\sqrt{25.9} = 5.3$ correct?

Solution

Method 1

To verify the calculation, apply the relationship $\sqrt{a} = b$.

Let $a = 25.9$
$\sqrt{a} = b$
$\sqrt{25.9} = 5.08920425...$
Since $5.08920425... \neq 5.3$ the calculation is incorrect.

Method 2

To verify the calculation, apply the relationship $b^2 = a$

Let $b = 5.3$
$b^2 = a$
$(5.3)^2 = 28.09$
Since $28.09 \neq 25.3$, the calculation is incorrect.