Estimating Square Roots

To estimate the square root of a non-perfect square, use the perfect squares above and below the given number.

Example

Using perfect squares, estimate the value of $\sqrt{104}$.

Solution

Step 1

Determine two perfect squares closest to the given number, and calculate their square roots.

- The closest perfect square smaller than 104 is 100. Its square root is 10.
  $\sqrt{100} = 10$ because $10 \times 10 = 100$.
- The closest perfect square larger than 104 is 121. Its square root is 11.
  $\sqrt{121} = 11$ because $11 \times 11 = 121$.

Step 2

Determine which of the values the non-perfect square is closest to, and approximate the square root.

Since 104 is between 100 and 121, its square root is between 10 and 11.

There are 21 numbers between 100 and 121, and four numbers between 100 and 104. The square root of 104 is $\frac{4}{21}$ of the distance between 10 and 11 or approximately 0.2

$\therefore \ \sqrt{104} \approx 10.2$

The estimated value of the square root of 104 is 10.2

Because non-perfect square roots are numbers that never end, a calculator only gives an approximate answer.

Example

Using a calculator, find the value of $\sqrt{20}$ rounded to the nearest tenth.

Solution

Each calculator is different, but generally, the square root of 20 can be calculated in one of two ways:

- Type the number, and then press $\boxed{\sqrt{}}$.

- Press $\boxed{\sqrt{}}$, type the number, and then press $\boxed{=}$.

Using either method, $\sqrt{20} = 4.472135955$.

The tenths place is one place after the decimal. Rounded to the nearest tenth, $\sqrt{20} = 4.5$

To estimate the square root of a decimal number, use the decimal equivalent to a known whole number perfect square.

Note the number of digits to the right of the decimal in a square number will be even.

Example

Using perfect squares, what is a reasonable estimate of the square root of 0.73?

Solution

Step 1

Determine two perfect square closest to the given number.

The closest perfect square smaller than 0.73 is 0.64.

Its square root is 0.8
$\sqrt{0.64} = 0.8$ because $0.8 \times 0.8 = 0.64$

The closest perfect square larger than 0.73 is 0.81

Its square root is 0.9
$\sqrt{0.81} = 0.9$ because $0.9 \times 0.9 = 0.81$

Step 2

Determine the square root between the two perfect squares.

The square root of 0.64 is 0.8, and the square root of 0.81 is 0.9

Therefore, the square root of 0.73 must be between 0.8 and 0.9

Since 0.73 is almost exactly between 0.64 and 0.81, it's approximate square root is 0.85.

A reasonable estimate of the square root of 0.73 is 0.85.